1.2: Linear Waves

An important feature of linear waves is that the dispersion relation captures the full system in Fourier space. That is, if the physical system takes the schematic form

$$D(i\frac{\partial}{\partial t}, -i\frac{\partial}{\partial x})\, u = 0\,, \quad \text{then} \quad D(\omega, k) = 0\,, \tag{4}$$

whose solutions are the branches $\omega = \omega(k)$. For stable waves, ω is real-valued for all real-valued k. There are two important velocities,

$$\text{Phase Velocity :} \quad c = \frac{\omega}{k}\,, \quad \text{and} \quad \text{Group Velocity :} \quad c_g = \frac{d\omega}{dk}\,. \tag{5}$$

For a dispersive wave system, they are different. The phase of the wave (e.g. a wave crest) propagates with velocity c, but the wave energy propagates with the velocity c_g. The wave energy E for each Fourier component is typically given by an expression of the form $E = F(k)|A|^2$. For instance, for water waves $E = g|A|^2/2$ where A is the surface elevation above the still-water depth.